This exam contains 9 pages (including this cover page) and 20 questions. The total number of points is 100. You have 75 minutes to complete the exam.

Read each question carefully. When specified, you must show **all** *necessary* work to receive full credit.

No calculator/phone/smartwatch allowed under any circumstances. Place these items in your bag, out of reach. Cheating of any kind will not be tolerated and will result in a grade of zero.

Question	Marks	Score	Question	Marks	Score
1	3		11	6	
2	3		12	10	
3	3		13	2	
4	3		14	5	
5	3		15	4	
6	3		16	5	
7	3		17	11	
8	3		18	8	
9	3		19	9	
10	3		20	10	
			Total	100	

1. (3 marks) True or False: The y-intercept of $f(x) = a(x-s)^2 + t$ is $y = t$.

    A. True   B. False

2. (3 marks) True or False: For two functions $f(x)$ and $g(x)$, if $f(g(x)) = x$ then $f(x)$ and $g(x)$ are inverses of each other.

    A. True   B. False

3. (3 marks) True or False: $\log_7 (56) = 8$.

    A. True   B. False

4. (3 marks) Fill in the blank:

$$\log\_ (243) = 5$$

5. (3 marks) Fill in the blank:

$$\log_5 (\text{\_\_\_\_}) = 4$$

6. (3 marks) Fill in the blank:

$$\log_4 (1024) = \text{\_\_\_\_}$$

For questions 7-10, choose the best answer. There is only one correct answer but you may choose up to *two*. If you choose two and one of the answers is correct, you will receive 1 mark.

7. (3 marks) If an initial amount of P dollars is invested in an account with an annual interest rate of r, compounded n times a year, which of the following represents the amount in the account after t years.

    A. $A(t) = P(1+r)^{nt}$ C. $A(t) = P\left(1+\dfrac{r}{n}\right)^{nt}$

    B. $A(t) = P\left(1+\dfrac{r}{n}\right)^{t}$ D. $A(t) = Pe^{rt}$

8. (3 marks) If $\log_9(x - 5) + \log_9(x + 3) = 1$, then

    A. $x = 6$ C. $x = 6$ or $x = -4$

    B. $x = -4$ D. None of these.

9. (3 marks) If $f(x) = \sqrt[7]{x + 2}$, then

    A. $f^{-1}(x) = x^7 + 2$ C. $f^{-1}(x) = x^7 - 2$

    B. $f^{-1}(x) = (x - 2)^7$ D. None of these.

10. (3 marks) If $f(x) = x^2 + 5$, then

    A. $f^{-1}(x) = \sqrt{x - 5}$ C. $f^{-1}(x) = \sqrt{x} - \sqrt{5}$

    B. $f^{-1}(x) = \sqrt{x} - 5$ D. None of these.

For questions $11 - 21$ show **all** *necessary* work to receive full credit. Circle or box your final answer.

As discussed, Questions 13-16 are all homework 7.

11. (6 marks) Fill in the table below:

$\log_a (xy) =$	$\log_a \left(\frac{x}{y} \right) =$	$\log_a (x^n) =$
$\log_a(1) =$	$\log_a(a) =$	$a^{\log_a(x)} =$

12. (10 marks) If a, r, s, t are all positive real numbers, then;

$f(x - t)$ is a _____ of $f(x)$ _____ units to the _____.

$f(x + t)$ is a _____ of $f(x)$ _____ units to the _____.

$af(x)$ is a _____ of $f(x)$ in the _____, by a scale factor of _____.

$-f(x)$ is a _____ of $f(x)$ in the _____.

$f(x) + s$ is a _____ of $f(x)$ _____ units _____.

$f(x) - s$ is a _____ of $f(x)$ _____ units _____.

$f(rx)$ is a _____ of $f(x)$ in the _____, by a scale factor of _____.

$f(-x)$ is a _____ of $f(x)$ in the _____.

13. (2 marks) Pete is deciding which account to invest his $3000 in. Below are the details of each account.

   - Account A offers a 4% annual interest rate, compounded monthly
   - Account B offers a 4% annual interest rate, compounded quarterly (1 quarter $=$ 3 months)

   Find two models, $A(t)$ and $B(t)$, that represent the amount in accounts A and B respectively after t years.

14. Food poisoning is often caused by E. coli bacteria. To test for the presence of E. coli in a pot of beef stew, a biologist performs a bacteria count on a small sample of the stew kept at 25°C. She determines the count is 5 units per millilitre and the number will double every 40 minutes.

   (a) (2 marks) Find the *hourly* growth factor a and find an exponential model $f(t) = Ca^t$ for the bacteria count in the beef stew.

   (b) (1 mark) Find the instantaneous growth rate r and find an exponential model $g(t) = Ce^{rt}$ for the bacteria count t hours later.

   (c) (2 marks) Using you model from part b), determine how many hours it would take for the amount of bacteria to reach 25 units.

15. (4 marks) If $25,000$ is invested at an interest rate of 7% per year, find the models that represent the amount of the investment $A(t)$ for the following compounding methods;

  Quarterly

  Monthly

  Daily

  Continuously

16. Internet World Stats reports that the number of internet users in China increased by 1024% from 2000 to 2008 (so the 8 year growth rate is 10.24 and the 8 year growth factor is 11.24). The number of internet users in 2000 was 22.5 million. Assume that the number of internet users increases exponentially.

   (a) (2 marks) Find the *yearly* growth factor a and find an exponential model $f(t) = Ca^t$ for the number of internet users in China after t years since 2000, where the number of users is measured in millions.

   (b) (1 mark) Find the instantaneous growth rate r and find an exponential model $g(t) = Ce^{rt}$ for the number of internet users in China after t years since 2000.

   (c) (2 marks) In what year would you expect the number of users in China to reach 450 million?

17. (11 marks) Consider the quadratic function $f(x) = -x^2 - 4x - 3$. Fill out the table and plot the graph, labeling the y intercept, roots and the vertex.

Domain:		Range:		y-intercept:		Roots:	
Minimum		Maximum:		Increasing:		Decreasing:	
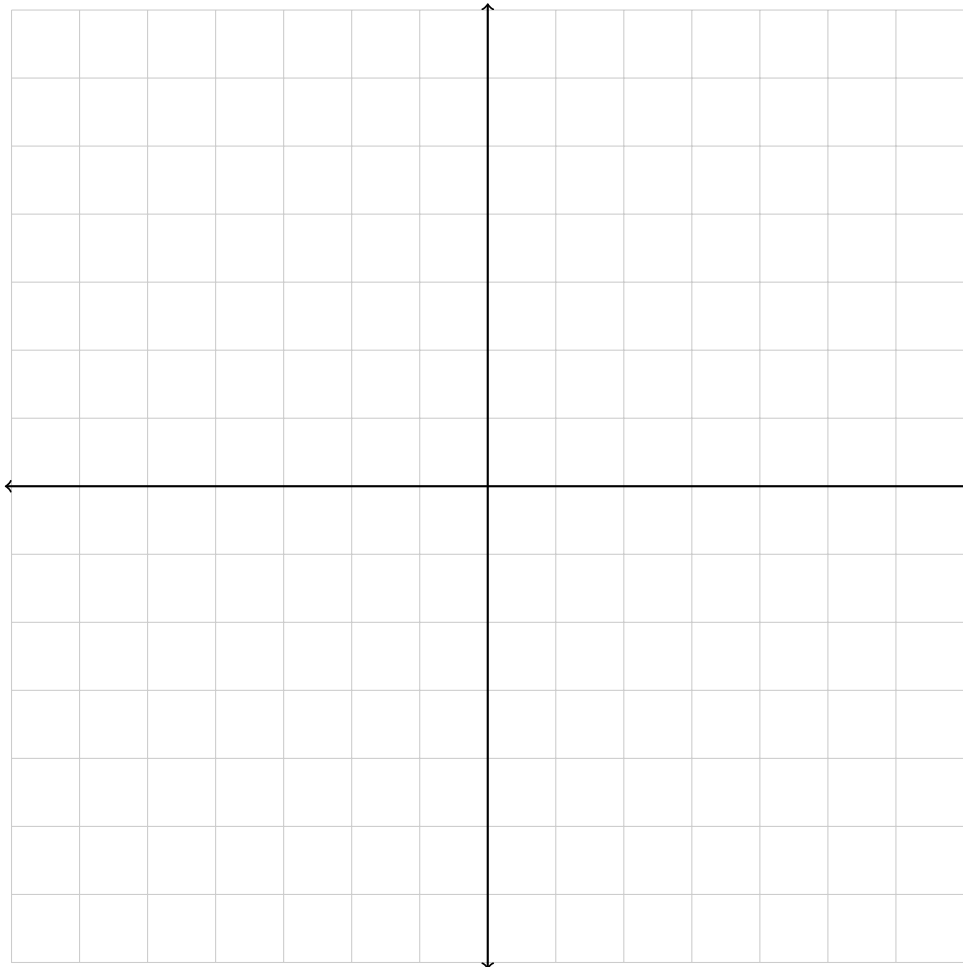
18. (8 marks) Determine the equation of the quadratic function whose graph is given below. Each square represents 1 unit.
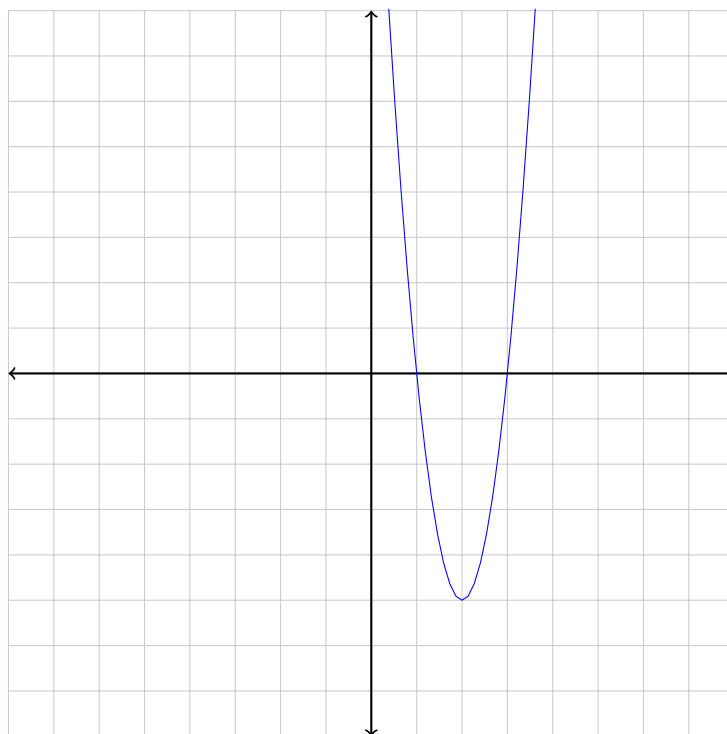
19. Express each of the following in terms of $\log(A)$, $\log(B)$ and $\log(C)$.

(a) (3 marks) $\log\left(\dfrac{A^2 B}{C}\right)$

(b) (3 marks) $\log\left(\left(\dfrac{A}{B}\right)^2\right)$

(c) (3 marks) $\log\left(B\sqrt{C}\right)$

20. Let $f(x) = 9x^2 - 30x + 25$ and $g(x) = x - 7$.

   (a) (2 marks) Describe $f(g(x))$ as a transformation of $f(x)$.

   (b) (8 marks) Compute $f(g(x))$ and simplify your answer fully.